Topic: Critical points

Question: Find the critical points of the function.

$$f(x) = x^2 - 10x + 2$$

Answer choices:

A $x = \frac{1}{5}$

B $x = 5$

C $x = -5$

D $x = -\frac{1}{5}$

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Solution: B

Critical points of a function $f(x)$ exist where the derivative of the function is equal to 0. To solve for critical points, we take the derivative of our function, set the derivative equal to 0, and then solve for x. It is possible for a function to have zero, one or multiple critical points.

Let's start by taking the derivative of our function $f(x) = x^2 - 10x + 2$.

$$f'(x) = 2x - 10$$

Now we can set the derivative equal to 0 and solve for x.

$$2x - 10 = 0$$

$$2x = 10$$

$$x = 5$$

The function $f(x) = x^2 - 10x + 2$ has one critical point at $x = 5$.

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Topic: Critical points

Question: Find the critical points of the function.

$$f(x) = -3x^3 + 16x + 6$$

Answer choices:

A $x = \pm\frac{16}{9}$

B $x = \pm\frac{9}{16}$

C $x = \pm\frac{3}{4}$

D $x = \pm\frac{4}{3}$

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Solution: D

Critical points of a function $f(x)$ exist where the derivative of the function is equal to 0. To solve for critical points, we take the derivative of our function, set the derivative equal to 0, and then solve for x. It is possible for a function to have zero, one or multiple critical points.

Let's start by taking the derivative of our function $f(x) = -3x^3 + 16x + 6$.

$$f'(x) = -9x^2 + 16$$

Now we can set the derivative equal to 0 and solve for x.

$$-9x^2 + 16 = 0$$

$$-9x^2 = -16$$

$$x^2 = \frac{16}{9}$$

$$x = \pm\sqrt{\frac{16}{9}}$$

$$x = \pm\frac{4}{3}$$

The function $f(x) = -3x^3 + 16x + 6$ has two critical points at $x = \pm\frac{4}{3}$.

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Topic: Critical points

Question: Find the critical points of the function.

$$f(x) = 4x^3 + \frac{6}{x}$$

Answer choices:

A $x = \pm\sqrt{2}$

B $x = \pm\dfrac{\sqrt{2}}{2}$

C $x = \pm\dfrac{\sqrt[4]{8}}{2}$

D $x = \pm\sqrt[4]{2}$

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Solution: C

Critical points of a function $f(x)$ exist where the derivative of the function is equal to 0. To solve for critical points, we take the derivative of our function, set the derivative equal to 0, and then solve for x. It is possible for a function to have zero, one or multiple critical points.

Let's start by taking the derivative of our function $f(x) = 4x^3 + \frac{6}{x}$.

$$f'(x) = 12x^2 - \frac{6}{x^2}$$

$$f'(x) = \frac{12x^4 - 6}{x^2}$$

Now we can set the derivative equal to 0 and solve for x.

$$\frac{12x^4 - 6}{x^2} = 0$$

$$12x^4 - 6 = 0$$

$$12x^4 = 6$$

$$x^4 = \frac{6}{12}$$

$$x^4 = \frac{1}{2}$$

$$x = \pm\sqrt[4]{\frac{1}{2}}$$

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$$x = \pm \frac{1}{\sqrt[4]{2}}$$

In order to rationalize the denominator, we'll rewrite the critical points as

$$x = \pm \frac{1}{2^{\frac{1}{4}}}$$

Then we'll multiply both the numerator and denominator by $2^{\frac{3}{4}}$ so that the denominator will become 2 when the exponents are added together.

$$x = \pm \frac{1}{2^{\frac{1}{4}}} \left(\frac{2^{\frac{3}{4}}}{2^{\frac{3}{4}}} \right)$$

$$x = \pm \frac{2^{\frac{3}{4}}}{2^{\frac{1}{4}+\frac{3}{4}}}$$

$$x = \pm \frac{2^{\frac{3}{4}}}{2}$$

$$x = \pm \frac{\left(2^3\right)^{\frac{1}{4}}}{2}$$

$$x = \pm \frac{8^{\frac{1}{4}}}{2}$$

$$x = \pm \frac{\sqrt[4]{8}}{2}$$

The function $f(x) = 4x^3 + \frac{6}{x}$ has two critical points at $x = \pm \frac{\sqrt[4]{8}}{2}$.

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